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Plane Geometry Problems With Solutions

The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS
UWA ACADEMY
FOR YOUNG MATHEMATICIANS

Plane geometry – Circles: Problems with some Solutions

1. Prove that for any triangle ABC , the perpendicular bisectors of the three sides AB , BC and CA are concurrent at O and deduce that $AO = BO = CO$, and hence O is the centre of a circle k of radius $R = AO$ that passes through each of the vertices A, B, C of $\triangle ABC$. (k, O, R are respectively the circumcircle, circumcentre and circumradius of $\triangle ABC$, and k is circumscribed about $\triangle ABC$.)

Solution. Draw $\triangle ABC$ and let the midpoints of sides AB , BC and CA be D, E and F , respectively. Draw the perpendicular bisectors from sides AB and BC to meet at a point O , and join OA to F (we must show OF is also a perpendicular bisector). Thus

$AD = BD$
 $\angle ODA = \angle ODB$
 OD is common
 $\therefore \triangle ODA \cong \triangle ODB$ by the SAS Rule
 $\therefore AO = BO$
Similarly $\triangle OEB \cong \triangle OEC$
 $\therefore BO = CO$

So now we have
 $AO = CO$
 $AF = CF$
 OF is common
 $\therefore \triangle OFA \cong \triangle OFC$ by the SSS Rule
 $\therefore \angle OFA = \angle OFC = 90^\circ$, since $\angle AFC$ is a straight angle (180°)

Thus, OF is the perpendicular bisector of AC , and hence the perpendicular bisectors of $\triangle ABC$ meet at O . Also, we showed that $AO = BO = CO$, so that A, B and C lie on a circle centred at O with radius $R = AO$.

2. Prove that the angle bisectors of a triangle ABC are concurrent, and that their common point I is equidistant from the sides of $\triangle ABC$.
(Let r be the common distance of I from the three sides of $\triangle ABC$. Then the circle $k(I, r)$ with centre I and radius r is inscribed in $\triangle ABC$, and $k(I, r), I, r$ are respectively the incircle, incentre and inradius of $\triangle ABC$.)

Solution. Draw $\triangle ABC$ and draw the angle bisectors from A and B to meet at I . Drop perpendiculars from I to points D, E and F , of the sides AB, BC and CA , respectively, and join I to C (we must show IC is also an angle bisector). Thus