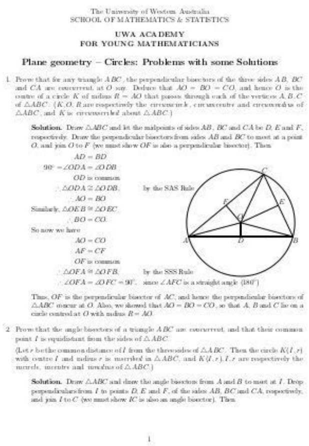

The University of Western Australia
SCHOOL OF MATHEMATICS & STATISTICS

UWA ACADEMY
FOR YOUNG MATHEMATICIANS

Plane geometry – Circles: Problems with some Solutions

1. Prove that for any triangle ABC, the perpendicular bisectors of the three sides AB, BC and CA are concurrent, at O say. Deduce that $AO = BO = CO$, and hence O is the centre of a circle K of radius $R = AO$ that passes through each of the vertices A, B, C of $\triangle ABC$. (K, O, R are respectively the circumcircle, circumcentre and circumradius of $\triangle ABC$, and K is circumscribed about $\triangle ABC$.)

   Solution. Draw $\triangle ABC$ and let the midpoints of sides AB, BC and CA be D, E and F, respectively. Draw the perpendicular bisectors from sides AB and BC to meet at a point O, and join O to F (we must show OF is also a perpendicular bisector). Then

   $AD = BD$
   $90° = \angle ODA = \angle ODB$
   OD is common
   $\therefore \triangle ODA \cong \triangle ODB$, by the SAS Rule
   $\therefore AO = BO$
   Similarly, $\triangle OEB \cong \triangle OEC$,
   $\therefore BO = CO$.
   So now we have
   $AO = CO$
   $AF = CF$
   OF is common
   $\therefore \triangle OFA \cong \triangle OFB$, by the SSS Rule
   $\therefore \angle OFA = \angle OFC = 90°$, since $\angle AFC$ is a straight angle ($180°$)

   Thus, OF is the perpendicular bisector of AC, and hence the perpendicular bisectors of $\triangle ABC$ concur at O. Also, we showed that $AO = BO = CO$, so that A, B and C lie on a circle centred at O with radius $R = AO$.

2. Prove that the angle bisectors of a triangle ABC are concurrent, and that their common point I is equidistant from the sides of $\triangle ABC$.

   (Let r be the common distance of I from the three sides of $\triangle ABC$. Then the circle $K(I, r)$ with centre I and radius r is inscribed in $\triangle ABC$, and $K(I, r)$, I, r are respectively the incircle, incentre and inradius of $\triangle ABC$.)

   Solution. Draw $\triangle ABC$ and draw the angle bisectors from A and B to meet at I. Drop perpendiculars from I to points D, E and F, of the sides AB, BC and CA, respectively, and join I to C (we must show IC is also an angle bisector). Then

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